Kriging the strain tensor based on geodetic, geotechnic and geological observations

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1. Introduction

Our interest is the mapping of strain in regions of recent deformation. Since neogene strain is typically very small, we need a proper stochastic integration of all available data, at a very bad signal to noise level. Depending on the scale varies sources of data are useful. Geodetic, geotechnic and geological data should be used together.

Geodetic data typically comes to us as adjusted point locations or heights at given times accompanied with a covariance matrix. Relevant geotechnic observations are tiltmeter and extensometer measurements giving local inclination changes and changes of the distance between inexactly specified points often on different sides of a fault. Geology provides, among other structural data, mapping, interpretation, and inclination of faults and their suspected possible directions of movement.

We model the area of interest as a set of discrete bodies with inner deformation described by an intrinsic random field of order $k = 1$ moving relatively to one another according to the constrains provided by the network of (major) faults. This approach allows to integrate all observations to a single stochastic model and to compute generalized covariances for all observations and any linear functionals of the deformation field. Since all these observations are invariant under the affine transformation, we can only estimate relative movements along the faults and the first derivative of translation, which is the strain tensor. The best linear predictor for the entries and linear functionals of the strain tensor (e.g. dilatation) is then provided by the kriging predictor. This method is somehow the opposite of gradient kriging, since we krige the gradients from the integrated observations.

2. Measurements of deformation

Small deformations of the rigid earth are of special interest in the neotectonics and engineering geology. Small deformations of the rigid earth can be observed geodetically by repeated observations of geodetic observation networks, by repeated single geodetic measurements like alignement, satellite laser ranging or very long baseline interferometry and by geotechnic observations performed by extensometers or tiltmeters. Since we typically only have a small amount of data and a very bad signal to noise ratio we need to integrate all available information into a mapping procedure of deformation.

Very important for mapping is the discrimination of localized relative displacement without deformation along major faults and homogenous ductile deformation or semihomogenous deformation in fault bundles. Informations about major fault can be obtained from structural observations.

2. Displacement and deformation

The deformation induced by a displacement field ζ_μ is described by the strain tensor (WELSCH 1982) $\mathcal{E}_{\mu\nu} = \frac{1}{2}(\zeta_{\mu,\,\nu}+\zeta_{\nu,\,\mu})$ and its compartments (dilatation $\mathcal{E}^{(D)}_{\mu\nu} = \frac{1}{3}\delta_{\mu\nu}\sum_\lambda \mathcal{E}_{\lambda\lambda}$ and deviator $\overset{(0)}{\mathcal{E}}_{\mu\nu} = \mathcal{E}_{\mu\nu} - \mathcal{E}^{(D)}$) or its other functionals like volume dilation $\Theta = \sum_\mu \mathcal{E}_{\mu\mu} \approx \operatorname{div}\zeta$, and its eigenvectors and eigenvalues $\mathcal{E}_I, \mathcal{E}_{II}, \mathcal{E}_{III}$. Here $\zeta_{\mu,\,\nu}$ denotes tensorial derivative of ζ (SCHMUTZER 1989). Additionally we are sometimes interested in datum dependent tensors like the rotation tensor ($D_{\mu\nu} = \overset{(0)}{\mathcal{E}}_{ij} = \frac{1}{2}(\zeta_{\mu,\,\nu} - \zeta_{\nu,\,\mu})$), shearing tensor and the displacement field itself $\zeta_\mu = x_\mu(\bar{x}_n u, t_1) - x_\mu(\bar{x}_n u, t_0)$. Here $x_\mu(\bar{x}_\nu, t_i)$ denotes the position material point $\bar{x}$ at time t_i. Since we are only concerned with small deformations and displacements we can the equation from linearized continuum mechanics. Note that most of the interesting quantities and its components are linear functionals of the displacement field.

3. Measurements seen as functionals

For small deformations the mentioned measurements can be linearized to linear functions of the displacement field: The geodetically observed difference of the position of the same body point $\mathbf{x}$ at two times (epochs) is a direct observation of the displacement, modeled by the functional of evaluation at $\mathbf{x}$: $e_\mathbf{x}\zeta := \zeta(\mathbf{x})$. An extensometer between two points $\mathbf{x}_a$ and $\mathbf{x}_b$ is linearized by:

$$E_{\mathbf{x}_a,\mathbf{x}_b}\zeta \approx \frac{\mathbf{x}_b - \mathbf{x}_a}{\|\mathbf{x}_b - \mathbf{x}_a\|}\left(\zeta(\mathbf{x}_b) - \zeta(\mathbf{x}_a)\right)$$

An tiltmeter measures the inclination of a line between two points $\mathbf{x}_a$ and $\mathbf{x}_b$ in direction $\mathbf{v}$:

$$I_{\mathbf{x}_a,\mathbf{x}_b,\mathbf{v}} = \frac{\mathbf{v}'}{\|\mathbf{x}_b - \mathbf{x}_a\|}(\zeta(\mathbf{x}_b) - \zeta(\mathbf{x}_a))\rho$$

Thus all measurements can be approximately seen as linear functionals of the displacement field.

4. Modeling the displacement field as random field

In order to apply kriging, we model the displacement field $\zeta(\mathbf{x})$ as a random vector field with covariance function $c(\mathbf{x},\mathbf{y}) = \operatorname{cov}(\zeta(\mathbf{x}),\zeta(\mathbf{y})) \in I\!R^{3\times 3}$ with a trend model

$$E[\zeta(\mathbf{x})] = \sum_{k=1}^{p} \beta_i f_i(\mathbf{x})$$

with known functions $f : R^3 \to R^3$ and unknown parameters $\beta_i \in I\!R$. We assume that blocks separated by major faults can move and rotate independently from each other. Thus we use a trend model like

$$f_i \in \left\{ \delta_{\mathrm{ib}(\mathbf{x})}\begin{pmatrix}1\\0\end{pmatrix},\ \delta_{\mathrm{ib}(\mathbf{x})}\begin{pmatrix}0\\1\end{pmatrix},\ \delta_{\mathrm{ib}(\mathbf{x})}\begin{pmatrix}x_2\\-x_1\end{pmatrix},\ i = 1,\ldots,n_{\mathrm{blocks}} \right\}$$

which allow every block be translated and infinitesimally rotated independently of each other. Here $b(\mathbf{x}) \in 1,\ldots,n_{\mathrm{blocks}}$ denotes the number of the block of $\mathbf{x}$ and δ_{ij} Kronecker's delta. In 3 dimensions the trend model would read:

$$f_i \in \left\{ \delta_{\mathrm{ib}(\mathbf{x})}\begin{pmatrix}1\\0\\0\end{pmatrix},\ \delta_{\mathrm{ib}(\mathbf{x})}\begin{pmatrix}0\\1\\0\end{pmatrix},\ \delta_{\mathrm{ib}(\mathbf{x})}\begin{pmatrix}0\\0\\1\end{pmatrix},\right.$$

$$\delta_{i\mathrm{b}(\mathbf{x})}\begin{pmatrix} x_2 \\ -x_1 \\ 0 \end{pmatrix}, \delta_{i\mathrm{b}(\mathbf{x})}\begin{pmatrix} x_3 \\ 0 \\ -x_1 \end{pmatrix}, \delta_{i\mathrm{b}(\mathbf{x})}\begin{pmatrix} 0 \\ x_3 \\ -x_2 \end{pmatrix}, i = 1, \ldots, n_{\text{blocks}} \Big\}$$

This trend can be constrained, when we know the possible directions of movement along the major faults. Since the strain tensor (which is the first derivative of ζ) should exists, we can assume that the variogram of the field is twice differentiable in the origin. From the standard models only the Gaussian and the Matern covariogram are differentiable in the origin. Using Gaussian models leads to artefacts in the method due to very strong assumptions on higher order derivatives. However in our examples we have good experience with a Matern model(STEIN 1999) isotropic in measurements and space (BOOGAART&SCHAEBEN 2002).

$$c_{\sigma^2,r}(\mathbf{x},\mathbf{y}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{\sigma^2}{2^{\nu-1}}\Gamma(\nu)\left(\frac{2\nu^{\frac{1}{2}}\|\mathbf{x}-\mathbf{y}\|}{r}\right)^{\nu}\mathcal{K}_\nu\left(\frac{2\nu^{\frac{1}{2}}\|\mathbf{x}-\mathbf{y}\|}{r}\right)$$

For locations on different blocks we assume the displacement to be uncorrelated, since it very untypical that the material on both sides of the fault is dragged in the same direction, and thus we set $c(\mathbf{x},\mathbf{y})$ to be 0 for any locations $\mathbf{x}$ and $\mathbf{y}$ on different blocks. The positive definiteness is guaranteed with this settings. The parameters of the covariogram were estimated by a generalization of the methods proposed in (BOOGAART&BRENNING 2001).

5. Kriging the displacement field

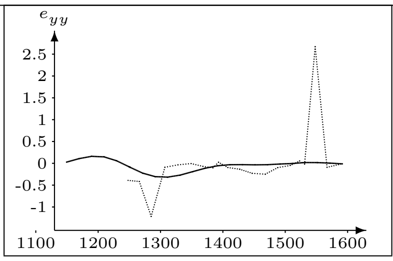
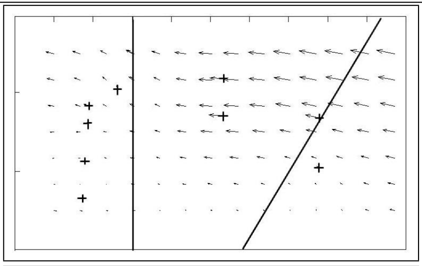
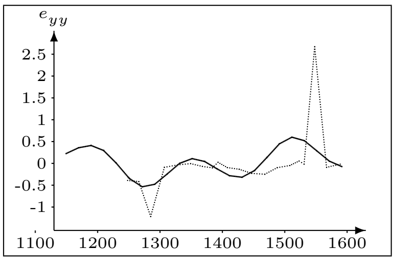
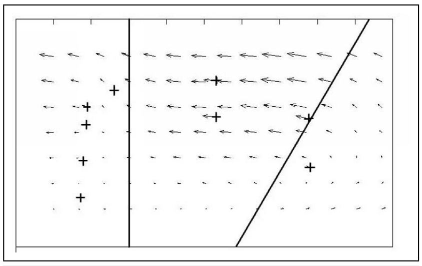
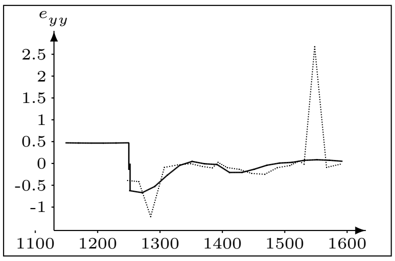
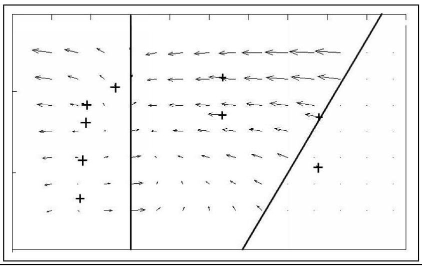
The problem in kriging the displacement field is that many of the observations are not really measurements of displacements but just measurements of linear functionals of the displacement field. Whenever the a linear functionals $L\zeta$ of the displacement fields exists in mean square sense its covariances and the trend functions can be calculated by (CHILES& DELFINER 1999):

$$\begin{aligned} \operatorname{cov}(L\zeta,\zeta(\mathbf{y})) &= L_{\mathbf{x}}c(\mathbf{x},\mathbf{y}) =: c(L,\mathbf{y}) \\ \operatorname{cov}(L\zeta,\tilde{L}\zeta) &= L_{\mathbf{x}}\tilde{L}_{\mathbf{y}}c(\mathbf{x},\mathbf{y}) =: c(L,\tilde{L}) \\ E[L\zeta] &= \sum_{i=1}^{n}\alpha_i L f_i =: \sum_{k=1}^{p}\beta_k f_k(L) \end{aligned}$$

We use the notations $c(L,\mathbf{y})$, $c(L,\tilde{L})$, and $f_k(L) := Lf_k$ as abbreviations for intuitive simplicity of resulting universal kriging equations. However we do not really observe the linear functionals directly. We observe quantities $\hat{L}_i = L_i\zeta + \varepsilon_i$, where the ϵ_i are correlated measurement errors. The variances $q_{ii} = \operatorname{var}(\varepsilon_i)$ and the covariances $q_{ij} := \operatorname{cov}(\varepsilon_i,\varepsilon_j)$ of the measurement errors are typically know from the measurement technique (e.g. specification of the extensometer) and from the adjustment of the geodetic network. Since the measurement error is independent of the deformation we have:

$$\operatorname{cov}(\hat{L}_i,\hat{L}_j) = c(L_i,L_j) + q_{ij}$$

Now we can place all these information into the kriging equation for the kriging of

Type	Distance dilatation in course of the pipeline in $[mm/m]$ 1968-1999; Solid - Calculated Dilatations; Dotted - Observed Dilatations	Estimated horizontal displacements 1968-1999.
I		
II		
III		

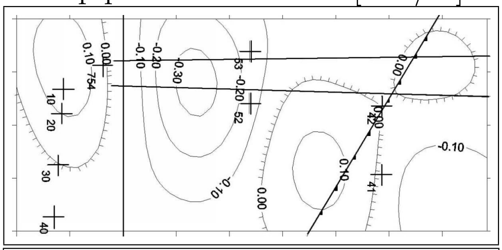
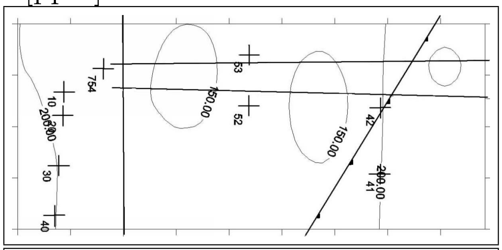
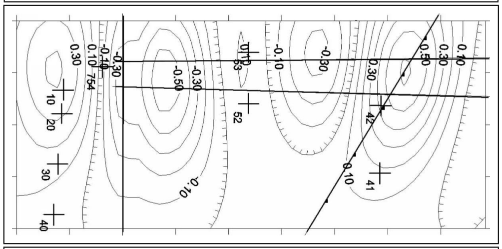
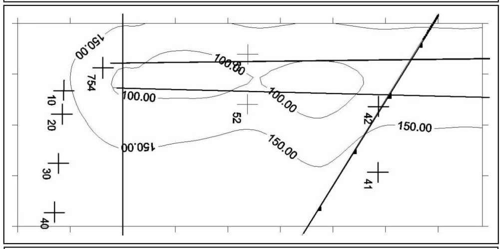
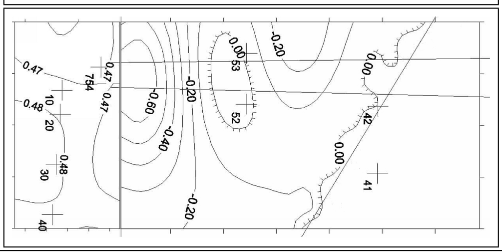
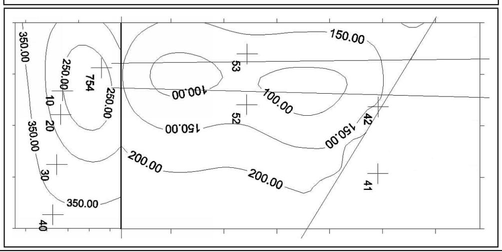
existing linear functional L of the displacement field:

$$\widehat{L\zeta} = \begin{pmatrix} z(\mathbf{x}_1) \\ \vdots \\ z(\mathbf{x}_n) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} c(\mathbf{x}_1,\mathbf{x}_1)+q_{11} & \cdots & c(\mathbf{x}_1,\mathbf{x}_n)+q_{1n} & f_1(\mathbf{x}_1) & \cdots \\ \vdots & \ddots & \vdots & \vdots & \ddots \\ c(\mathbf{x}_n,\mathbf{x}_1)+q_{n1} & \cdots & c(\mathbf{x}_n,\mathbf{x}_n)+q_{nn} & f_1(\mathbf{x}_n) & \cdots \\ f_1(\mathbf{x}_1)' & \cdots & f_1(\mathbf{x}_n)' & 0 & \cdots \\ \vdots & \ddots & \vdots & \vdots & \ddots \\ f_p(\mathbf{x}_1)' & \cdots & f_p(\mathbf{x}_n)' & 0 & \cdots \end{pmatrix}^{-1} \begin{pmatrix} L_{\mathbf{x}}c(\mathbf{x},x_1) \\ \vdots \\ L_{\mathbf{x}}c(\mathbf{x},x_n) \\ L_{\mathbf{x}}f_1(\mathbf{x})' \\ \vdots \\ L_{\mathbf{x}}f_p(\mathbf{x})' \end{pmatrix}$$

It can be shown that this is the best linear unbiased predictor of $L\zeta$ with the same arguments as universal kriging provides the best linear unbiased predictor.

6. Application to the hydroelectric power plant Hohenwarte II

The hydroelectric power plant Hohenwarte II was built in the years of 1965/66. It is with 320MW the biggest plant on the river Saale. The difference in altitude between upper and the lower water reservoir is about 300m and the pipeline between the these reservoirs is about 600m long. Caused by the geology of the Saale-valley the area is divided by faults into eight blocks, which are separately deformed and translated. To protect the buildings of the power plant a geodetic network around the power plant has been observed since 1967. Nine points of this network are located in the area around the pipeline and the powerhouse. Additional to the geodetic network the pipeline are monitored by 82 distance measurements. The results of this monitoring was used to describe the movement around

Type	Estimated dilatation in course of the pipeline 1968-1999. [mm/m]	Kriging error for this dilatation [ppm]
I		
II		
III		

the power plant and to detect areas with critical strain. This analyzes have shown marked translations in the slope-area and small translations in the valley-area(Arcadis GmbH 199?)(Schmidt 2001). The results of the standard analysis, the calculated distance dilatations in course of the pipeline, based on the nine geodetic points and the results of the direct measurements of the pipeline itself have shown clear differences. On the one hand there are extrema calculated from the strain analysis are not observed and on the other hand there are extrema, which would be observed, but not calculated. That's why it's suggested to use the direct measurements to improve the description of strain in the area around the pipeline.

Prerequisite of any strain analysis is a block model for the area of interest. Practical methods for modeling an testing of models are shown in (Caspary 1987) and (Casparay&Borruta 1986). There are two possible models for Hohenwarte II. First the models without considering any geological structures and second the models with several homogenous blocks, whereby due to the less number of geodetic observations only a model with two blocks is possible. The model 1 is used for the strain analysis based only on the geodetic points, and the model 2 is used for a conjoint strain analysis. For Hohenwarte II there are three types of evaluations:

- Model I: model with one block model only with geodetic observations
- Model II: model with one block with geotechnic and geodetic observations
- Model III: model with separated blocks with geotechnic and geodetic observations

7. Results

If the results of the strain analysis and direct measurement, are compared to each other, then the objective of integration of geodetic and geotechnic observations, the improvement of strain parameters, is obviously to shown. In contrary to the ordinary kriging of the displacement field only with geodetic observations, the results of the the conjoint strain analysis based on our method of kriging of linear functionals for geodetic and geotechnic observations match better to the direct measurement. The additional analysis with model 2 results in further improvement, as the translations of two blocks against each other and deformations within the block will be separated.

The extremes of the distance dilatation in direction of the pipeline are around faults and near the border between the valley block and the mountainside. The deformations near the border are true internal deformations because there are no faults in this area. The calculation with blockmodelling in the middle block the deformations changes only a little, only the extrema are increasing. The maximum deformation in the upper part is therefore no longer interpreted as a deformation but as relative translation of this blocks. For the calculations of the middle block the model 2 seems to be the better choice. This model seems also be better for the valley block, but the additional informations about the translation of this block is not reliable. Even so the mapped big extensions in the valley block is probably an artefact of the method, because the number and the location of the points are less suitable for the calculation of distance extension in direction of the pipeline. Therefore even the big extensions are under the 2σ-limit of the kriging error.

8. References

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